A Numerical Simulation of Surface Waves, Wave-Current Interaction, and Langmuir Circulations

Y. Fujiwara[†], YU Yoshikawa[†], and Y. Matsumura[‡]

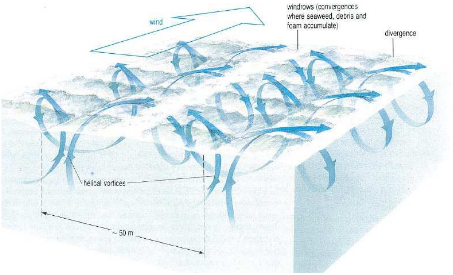
†: Kyoto University, ‡: University of Tokyo (Japan)

The 10th International Workshop on Modeling the Ocean

My Interests...

- Gravitational, Baroclinic, and Frontal Instabilities
  - "Transverse roll convection in horizontal plane Couette flow" (Yoshikawa and Akitomo, JFM2003)
  - "The subpolar front of the Japan/East Sea. Part III: Completing roles of frontal dynamics and atmospheric forcing in drivng ageostrophic verical circulations and subduction" (Yoshikawa, Lee and Thomas, JPO2012)
- Wind-Driven Flow and Mixing
  - "A surface velocity spiral observed with ADCP and HF radar in the Tsushima strait (Yoshikawa, Matsuno, Marubayashi and Fukudome. JGR2007)
  - "Seasonal variations in the speed factor and deflection angle of the wind-driven surface flow in the Tsushima strait (Yoshikawa and Masuda, JGR2009)
  - "Scaling surface mixing/mixed layer depth under stabilizing buoyancy flux (Yoshikawa, JPO2015)
- Langmuir Circulations (Wave-Current Interaction)

Langmuir Circulation (LC)



(Brown et al. 1989)

(Sullivan and McWilliams 2012)

- LC dominates vertical mixing when $La = (U_*/U_S)^{1/2} <$ is small. (e.g., $La <$ 0.3).
- LC has global impact on MLD. (e.g., Belcher et al. 2012)

Craik and Leibovich Theory

- LC is driven by wave-current interaction.
- The interaction can be represented as the Vortex Forcing (VF).

NS Equation

$$\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{\omega} \times \boldsymbol{u} = -\nabla \left(p/\rho_0 + |\boldsymbol{u}|^2/2 \right) - \boldsymbol{g} + \nu \nabla^2 \boldsymbol{u}$$

$\Downarrow$ $\boldsymbol{u} = \overline{\boldsymbol{u}}$ (mean flow comp.) $+ \boldsymbol{u}'$ (wave comp.)

$\Downarrow$ complicated math. + several assumptions

CL Equation (Craik and Leibovich 1976)

$$\frac{\partial \overline{\boldsymbol{u}}}{\partial t} + \overline{\boldsymbol{\omega}} \times \overline{\boldsymbol{u}} = -\nabla \left(\overline{p}/\rho_0 + |\overline{\boldsymbol{u}}|^2/2 + \Pi \right) - \boldsymbol{g} + \nu \nabla^2 \overline{\boldsymbol{u}} + \underbrace{\boldsymbol{u}_S \times (\overline{\boldsymbol{\omega}})}_{\text{Vortex Force}}$$

$$\boldsymbol{u}_S = \frac{1}{T} \int_0^T \boldsymbol{x}_w \frac{\partial \boldsymbol{u}_w}{\partial \boldsymbol{x}} dt$$: Stokes Drift Velocity

- VF can be interpreted as tilting of vortex by Stokes drift velocity
  ⇐Lagrangian view
  VF is more strictly derived in Generalized Lagrangian Mean (GLM) equations.(e.g., Andrews and McIntyre 1978, Leibovich 1980)

- VF induces LC through instability mechanism (CL2 mechanism)

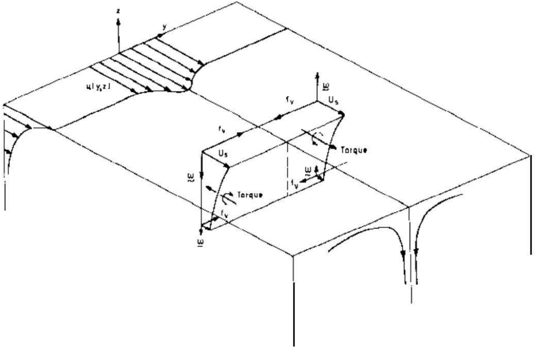


Figure 3 Sketch illustrating the CL 2, or instability mechanism of Langmuir-circulation generation. The Stokes drift is horizontal, but decays in depth. Streamwise vorticity is induced by the Stokes-drift rotation of vertical vorticity associated with spanwise perturbations of the current. Variations of vortex force caused by the current perturbation create torques leading to overturning.

- Many simulations with VF as an external forcing (without resolving wave itself) successfully reproduce observed features of LCs.
  (e.g., Skyllingstad and Denbo 1995; McWilliams et al 1997; Noh et al. 2004)
  - CL2 mechanism is likely to generate LC.
  - Validity of the VF expression is assumed.

- No direct verification (experimental validation) of the VF expression.
  - Field experiments: × Vorticity
  - Laboratory experiments: Side-wall effects, short-fetch, etc.
  - Numerical experiments: × Deep water waves

⇒Ongoing discussions...

The Present Approach : Direct Numerical Modelling

- Free-Surface Nonhydrostatic Ocean Model
  (KINACO, developed by Dr. Matsumura)
- – Nonhydrostatic pressure under the free surface (Casulli and Zanolli 2002)
- – Efficient Poisson/Helmholtz solver (Matsumura and Hasumi 2008)

- Direct numerical simulation of
- – deep water waves (Swell)
- – wave-current (wind-driven flow) interaction
- – Langmuir circulation?

Governing Equations

$$\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{\omega} \times \boldsymbol{u} = -\nabla \left(p/\rho_0 + |\boldsymbol{u}|^2/2 \right) - \boldsymbol{g} + \nu \nabla^2 \boldsymbol{u} + \boldsymbol{F}$$

$$\nabla \boldsymbol{u} = 0$$

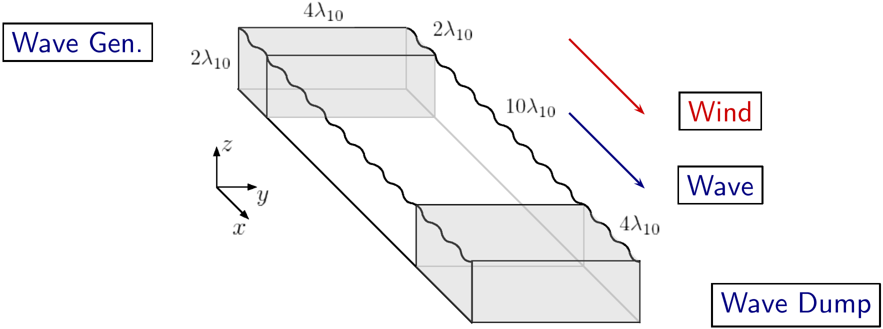
$$\frac{\partial \eta}{\partial t} + \boldsymbol{u}_H \cdot \nabla_H \eta = w(z = \eta) + F_\eta$$

$$\rho_0 = 1020 \text{ kg m}^{-3}, \quad \nu = 0.01 \text{ m}^2 \text{ s}^{-1}, \quad f = 0$$

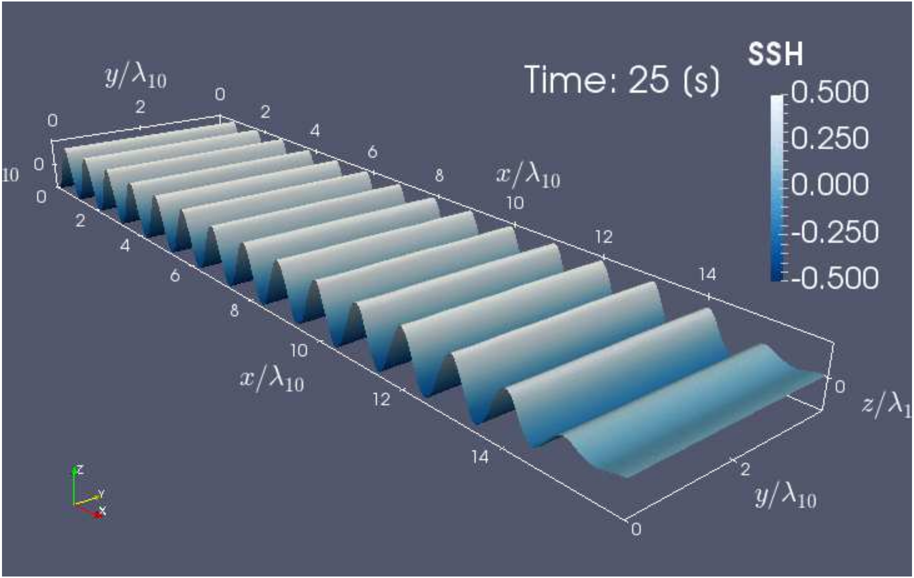
Model Configuration

A Wave Tank Experiment

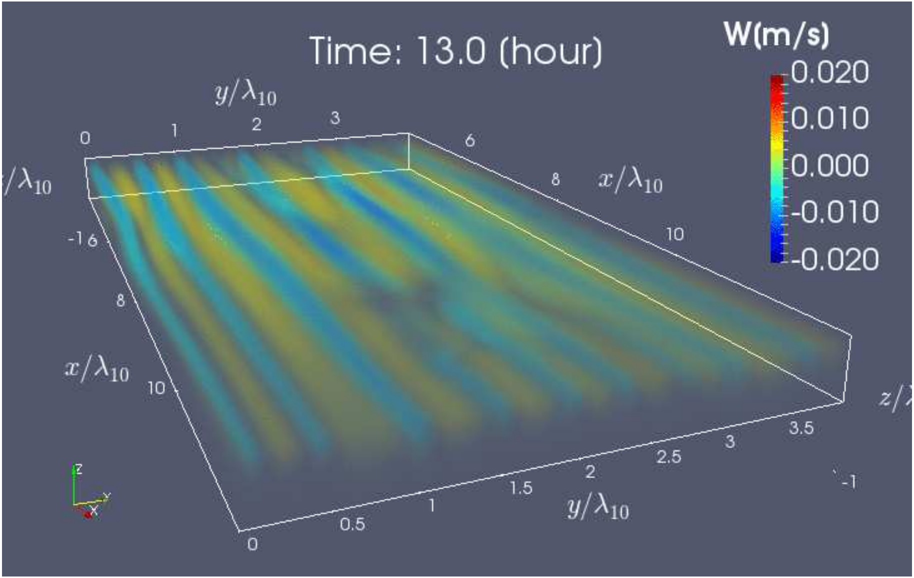
- Wave Generator + Dumper
  $A_{wave} = 0.5$ m, $T_{wave} = 10$ s
- Periodic in y direction
- Wind stress (0.02 N/m^2)
- Solid bottom
- Dimension: $16 \times 4 \times 2\ \lambda_{10}$



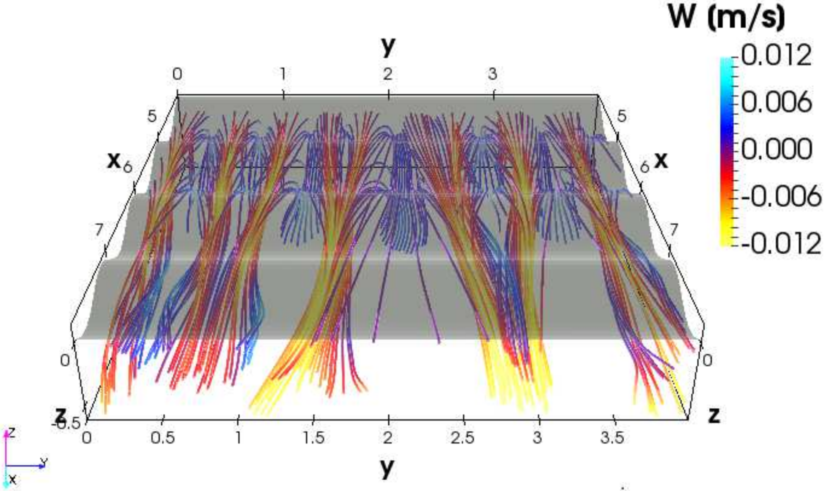
Simulated Waves



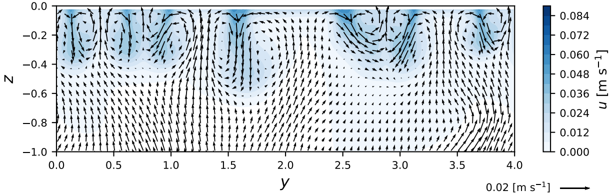
Simulated Flows (after averaging over the wave period)



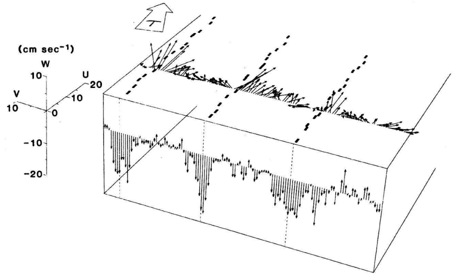
Simulated Flows (after averaging over the wave period)



Simulated



Observed

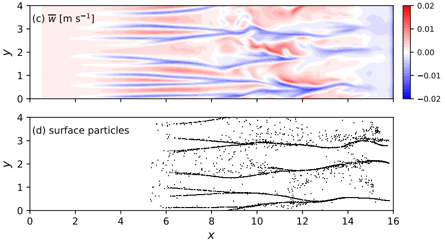


(Weller et al. 1985)

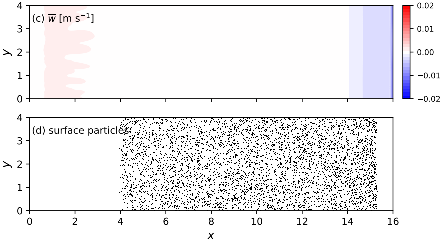
- Successfully reproduces observed features.

Wave and Wind Effects

With Wave



Without Wave



No LC-like circulation if

- waves are absent
- wind blows up-wave direction

$\Rightarrow$Simulated circulation is LC.

Vorticity Analysis

Wave-Averaged Vorticity Equation

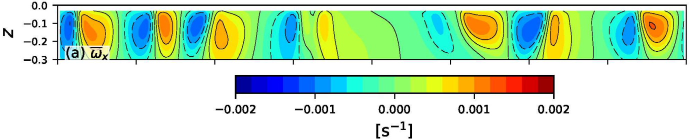
$$\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{\omega} \times \boldsymbol{u} = -\nabla \left(p/\rho_0 + |\boldsymbol{u}|^2/2 \right) - \boldsymbol{g} + \nu \nabla^2 \boldsymbol{u}$$

$$\Downarrow \quad \boldsymbol{u} = \overline{\boldsymbol{u}} + \boldsymbol{u}'$$

$$\Downarrow \quad \overline{\boldsymbol{\omega} \times \boldsymbol{u}} = \overline{\boldsymbol{\omega}} \times \overline{\boldsymbol{u}} + \overline{\boldsymbol{\omega}' \times \boldsymbol{u}'}, \quad \overline{|\boldsymbol{u}|^2/2} = |\overline{\boldsymbol{u}}|^2/2 + \overline{|\boldsymbol{u}'|^2}/2$$

$$\frac{\partial \overline{\boldsymbol{\omega}}}{\partial t} = \nabla \times \left(\overline{\boldsymbol{u}} \times \overline{\boldsymbol{\omega}} \right) + \underbrace{\nabla \times \left(\overline{\boldsymbol{u}' \times \boldsymbol{\omega}'} \right)}_{\text{wave effects}} + \nu \nabla^2 \overline{\boldsymbol{\omega}}$$

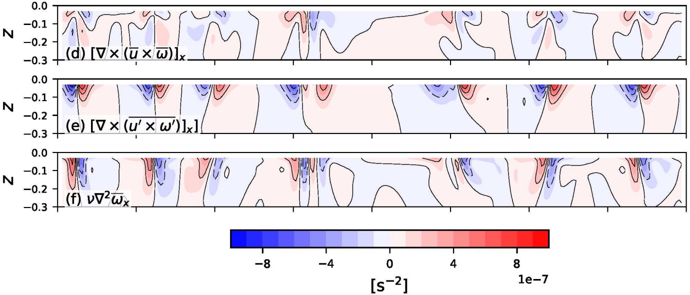
$$\overline{\omega}_x$$



$$\left[\nabla\times(\overline{\boldsymbol{u}}\times\overline{\boldsymbol{\omega}})\right]_x$$

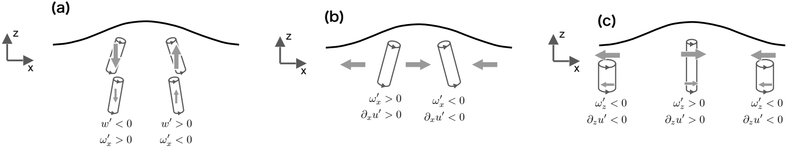
$$\left[\nabla\times(\overline{\boldsymbol{u}'\times\boldsymbol{\omega}'})\right]_x$$

$$\nu\nabla^2\overline{\omega}_x$$



$$\left[\nabla\times\left(\overline{\boldsymbol{u}'\times\boldsymbol{\omega}'}\right)\right]_x = -\frac{\partial\overline{u'\omega'_x}}{\partial x} - \frac{\partial\overline{v'\omega'_x}}{\partial y} - \frac{\partial\overline{w'\omega'_x}}{\partial z} + \overline{\omega'_x\frac{\partial u'}{\partial x}} + \overline{\omega'_y\frac{\partial u'}{\partial y}} + \overline{\omega'_z\frac{\partial u'}{\partial z}}$$

$$- \frac{\partial \overline{w' \omega'_x}}{\partial z} \qquad + \overline{\omega'_x \frac{\partial u'}{\partial x}} \qquad + \overline{\omega'_z \frac{\partial u'}{\partial z}}$$



Rectified tilting/streching/shrinking of background vorticity by waves

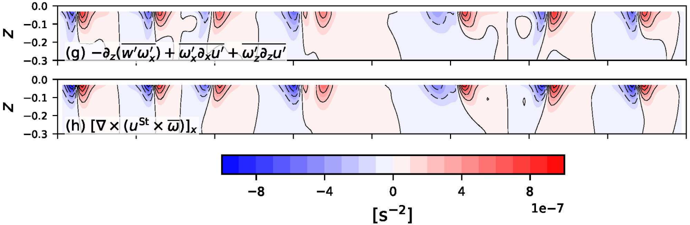
Comparison with VF

$$\boxed{\text{NS:}}\quad \frac{\partial \overline{\omega}}{\partial t} = \nabla \times (\overline{\boldsymbol{u}} \times \overline{\boldsymbol{\omega}}) + \nabla \times (\overline{\boldsymbol{u}' \times \boldsymbol{\omega}'}) + \nu \nabla^2 \overline{\boldsymbol{\omega}}$$

$$\boxed{\text{CL:}}\quad \frac{\partial \overline{\omega}}{\partial t} = \nabla \times (\overline{\boldsymbol{u}} \times \overline{\boldsymbol{\omega}}) + \nabla \times (\boldsymbol{u}_S \times \overline{\boldsymbol{\omega}}) + \nu \nabla^2 \overline{\boldsymbol{\omega}}$$

$\boxed{\nabla \times (\overline{\boldsymbol{u}' \times \boldsymbol{\omega}'})}$

$\boxed{\nabla \times (\boldsymbol{u}_S \times \overline{\boldsymbol{\omega}})}$



- Torque of wave-induced Reynolds stress $\simeq$ Torque of VF.
  $\Rightarrow$Experimental validation of VF.

Summary

The Present Study

- Direct numerical simulation of
  - deep water waves (+ wind-driven flow)
  - wave-current interaction
  - Langmuir circulation
- Eulerian view of wave-current interaction
  - Rectified vorticity tilting/shrinking/stretching by waves
- Simulated wave-current interaction $\simeq$ VF
  Note: assumptions in CL are all valid in the present simulation.

Future Work

- More general cases
  (large amplitude, phase speed $\simeq$ current speed, $\cdots$)

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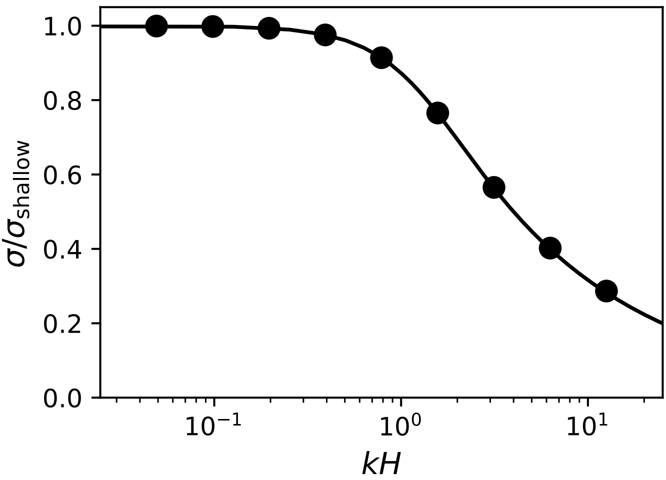
1.0
0.8
0.6
0.4
0.2
0.0
$\sigma/\sigma_{\mathrm{shallow}}$
10^{-1}
10^{0}
10^{1}
kH